On dual description of the $OSp(N|2m)$ sigma models

Based on M. Alfimov, B. Feigin, B. Hoare and A. Litvinov, arXiv:2003.xxxxx

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Motivation

- The integrability-preserving deformations of $O(N)$ sigma models are known to admit the dual description in terms of a coupled theory of bosons and Dirac fermions with exponential interactions of the Toda type (Fateev, Onofri, Zamolodchikov'93, Fateev'04, Litvinov, Spodyneiko'18).

- On the other hand, there are known examples of the integrable superstring theories, such as type IIB $AdS_5 \times S^5$ (dual to $\mathcal{N} = 4$ SYM) and others, which also have integrable deformations.

- Our strategic goal is to build a similar dual description for the deformed $AdS_5 \times S^5$ type IIB superstring (Arutyunov, Frolov et al.) and, possibly, other theories of this type.

- There are three major problems on this way:
  1. Incorporate the fermionic degrees of freedom into the construction of dual theory.
  2. Adapt the whole construction to describe the sigma models with non-compact target space.
  3. The superstring theory possesses the reparametrization symmetry and requires gauge fixing, which makes us include this symmetry into the dual description.

- In the present work we address the first problem generalizing the dual description of the deformed $O(N)$ sigma models to account for the $OSp(N|2m)$ sigma models.

The undeformed $OSp(N|2m)$ sigma model

- The $OSp(N|2m)$ sigma model is given by the symmetric space sigma model on the supercoset
$$\frac{OSp(N|2m)}{OSp(N-1|2m)} .$$
- The action for the supergroup-valued field $g \in OSp(N|2m)$ is
$$\mathcal{S}_0 = -\frac{R^2}{2} \int d^2x \, \mathrm{STr}[J_+ P J_-] ,$$
where $J_\pm = g^{-1}\partial_\pm g$ takes values in the Grassmann envelope of the Lie superalgebra $\mathfrak{osp}(N|2m;\mathbb{R})$ and STr is the invariant bilinear form.
- We are considering the symmetric space with the $\mathbb{Z}_2$ grading
$$\mathfrak{g} \equiv \mathfrak{osp}(N|2m;\mathbb{R}) = \mathfrak{g}^{(0)} \oplus \mathfrak{g}^{(1)} , \quad \mathfrak{g}^{(0)} = \mathfrak{osp}(N-1|2m;\mathbb{R})$$
and P being the projector onto the grade 1 subspace.
- This model is quantum integrable and has the following rational S-matrix (Saleur, Wehefritz-Kaufmann'01)
$$\check{S}^{j_2 j_1}_{i_1 i_2}(\theta) = \sigma_1(\theta) E^{j_2 j_1}_{i_1 i_2} + \sigma_2(\theta) P^{j_2 j_1}_{i_1 i_2} + \sigma_3(\theta) I^{j_2 j_1}_{i_1 i_2} ,$$
where
$$\sigma_1(\theta) = -\frac{2i\pi}{(N-2m-2)(i\pi-\theta)}\sigma_2(\theta) , \quad \sigma_3(\theta) = -\frac{2i\pi}{(N-2m-2)\theta}\sigma_2(\theta) .$$

Trigonometric $OSp(N|2m)$ R-matrix

- Besides rational solution, the Yang-Baxter equation

$$\check{R}^{k_2k_1}_{i_1i_2}(\mu)\check{R}^{k_3j_1}_{k_1i_3}(\mu+\rho)\check{R}^{j_3j_2}_{k_2k_3}(\rho)=\check{R}^{k_3k_2}_{i_2i_3}(\mu)\check{R}^{j_3k_1}_{i_1k_3}(\mu+\rho)\check{R}^{j_2j_1}_{k_1k_2}(\rho)$$

has the trigonometric solution (Bazhanov, Shadrikov'87) with the parameter q.

- Introducing the parametrization

$$q=e^{2i\pi\lambda}\ ,\quad \mu=(N-2m-2)\lambda\theta\ ,$$

we observe that for $\lambda=0$ it is consistent with the rational limit and in the special point $\lambda=\frac{1}{2}$ the $\check{R}$-matrix demonstrates an interesting behaviour.

- It becomes proportional to the S-matrix, corresponding to the scattering of the free theory consisting of $\frac{N}{2}$ Dirac fermions and m superghost particles in the case of even N and the same plus one boson in the case of odd N.

Special point of the $OSp(N|2m)$ R-matrix

- The $O(3)$ example with $N=3$, $m=0$ at $\lambda=\frac{1}{2}$:

$$\frac{\check{R}^{j_2 j_1}_{i_1 i_2}}{\check{R}^{22}_{22}} = \begin{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{pmatrix} + \mathcal{O}\left(\lambda - \frac{1}{2}\right) .$$

- The $OSp(1|2)$ example with $N=1$, $m=2$ at $\lambda=\frac{1}{2}$:

$$\frac{\check{R}^{j_2 j_1}_{i_1 i_2}}{\check{R}^{22}_{22}} = \begin{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{pmatrix} + \mathcal{O}\left(\lambda - \frac{1}{2}\right) .$$

The deformed O(3) dual model

- In the work (Fateev, Onofri, Zamolodchikov'93) there was studied the dual description of the sigma model with the metric ($\lambda = \nu + \mathcal{O}(\nu^2)$)

$$ds^2 = \frac{\kappa}{\nu}\left(\frac{dr^2}{(1-r^2)(1-\kappa^2 r^2)} + \frac{1-r^2}{1-\kappa^2 r^2}d\phi^2\right) .$$

  In the other limit $\lambda \to \frac{1}{2}$ the special integrable perturbation of the Sine-Liouville theory ($\lambda = \frac{1}{2} - \frac{b^2}{2} + \mathcal{O}(b^4)$)

$$\begin{aligned}\mathcal{L} = \frac{(\partial_\mu \Phi)^2}{8\pi} + \frac{(\partial_\mu \varphi)^2}{8\pi} - \\ - \frac{m}{4}\left(e^{b\Phi + i\beta\varphi} + e^{b\Phi - i\beta\varphi} + e^{-b\Phi + i\beta\varphi} + e^{-b\Phi - i\beta\varphi}\right) - \\ - \frac{m^2}{32\pi b^2}\left(e^{2b\Phi} - 2 + e^{-2b\Phi}\right) , \quad \beta = \sqrt{1+b^2} .\end{aligned}$$

  The sigma model coupling constant in the regime $b \to \infty$ is $\nu = \frac{2}{b^2} + \mathcal{O}\left(\frac{1}{b^4}\right)$.

- Using the Coleman-Mandelstam boson-fermion duality (Coleman'75, Mandelstam'75) $(\partial\varphi)^2/(8\pi) \to i\bar{\psi}\gamma^\mu\partial_\mu\psi$, $e^{\pm i\beta\varphi} \to \bar{\psi}(1 \pm \gamma_5)\psi$, we obtain

$$\begin{aligned}\mathcal{L} = \frac{(\partial_\mu \Phi)^2}{8\pi} + i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{\pi b^2}{2(1+b^2)}(\bar{\psi}\gamma^\mu\psi)^2 - \\ - m\bar{\psi}\psi\cosh(b\Phi) - \frac{m^2}{8\pi b^2}\sinh^2(b\Phi) .\end{aligned}$$

Building of the dual model

Guiding principles to look for the dual description (Litvinov, Spodyneiko'18)

1. The theory with the S-matrix as above has to be renormalizable (at least 1-loop). In the case of the deformed $O(3)$ it can be checked by the RG flow of the "sausage" metric.

2. The dual theory is found as an integrable perturbation from the special point of the S-matrix and is determined by the set of screening charges, which commute with the integrals of motion in the leading order in the mass parameter

$$\left[I_k^{\text{free}}, \int e^{(\boldsymbol{\alpha}_r, \phi)} dz \right] = 0 \,.$$

   In the case of the deformed $O(3)$ they are $e^{b\Phi + i\beta\varphi}$, $e^{b\Phi - i\beta\varphi}$, $e^{-b\Phi + i\beta\varphi}$ and $e^{-b\Phi - i\beta\varphi}$.

3. Our model is an integrable deformation of the CFT, based on the coset

$$\frac{\widehat{\mathfrak{osp}}(N|2m)_w}{\widehat{\mathfrak{osp}}(N-1|2m)_w} \,.$$

   Again, in the $O(N)$ case they are $\widehat{\mathfrak{so}}(N)_w / \widehat{\mathfrak{so}}(N-1)_w$.

The Yang-Baxter deformation of the $OSp(N|2m)$ sigma model

- The action for the Yang-Baxter deformed model is (Klimcik'02,Delduc'13)

$$\mathcal{S}_\eta = \int d^2x\, \mathcal{L}_\eta = -\frac{\eta}{2\nu} \int d^2x\ \mathrm{STr}[J_+ P \frac{1}{1-\eta \mathcal{R}_g P} J_-] ,$$

  where η is the deformation parameter and ν is the sigma model coupling.
- The operator $\mathcal{R}_g$ is defined in terms of an operator $\mathcal{R} : \mathfrak{g} \to \mathfrak{g}$ through

$$\mathcal{R}_g = \mathrm{Ad}_g^{-1}\, \mathcal{R}\, \mathrm{Ad}_g ,$$

  with $\mathcal{R}$ an antisymmetric solution of the (non-split) modified classical Yang-Baxter equation

$$[\mathcal{R}X, \mathcal{R}Y] - \mathcal{R}([X, \mathcal{R}Y] + [\mathcal{R}X, Y]) = [X, Y] ,$$
$$\mathrm{STr}[X(\mathcal{R}Y)] = -\mathrm{STr}[(\mathcal{R}X)Y] , \quad X, Y \in \mathfrak{g} .$$

- In terms of coordinates on the target superspace

$$\mathcal{L}_\eta = (G_{MN}(z) + B_{MN}(z))\, \partial_+ z^N \partial_- z^M , \quad z^M = (x^\mu, \psi^\alpha) ,$$

  where $G_{MN} = (-1)^{MN} G_{NM}$ and $B_{MN} = -(-1)^{MN} B_{NM}$.
- We explicitly calculated $G_{MN}(z)$ and $B_{MN}(z)$ in the range of parameters $N = 1, \ldots, 8$ and $m = 1, 2, 3$.

Ricci flow

- Substituting the metric and Kalb-Ramond field of the deformed $OSp(N|2m)$ sigma model for $m = 1$ with $N = 1, \ldots, 6$ into the Ricci flow equation

$$R_{MN} + \frac{d}{dt}E_{MN} + (\mathcal{L}_Z E)_{MN} + (dY)_{MN} = 0\,, \quad E_{MN} = G_{MN} + B_{MN}\,.$$

  we indeed find ($t \sim \log \Lambda_{UV}$)

$$\frac{d\nu}{dt} = 0\,, \quad \frac{d\eta}{dt} = -\nu(N - 2m - 2)(1 + \eta^2)\,.$$

  which is the natural expectation for general N and m. It agrees with the known result for $m = 0$ (Squellari'14, Litvinov, Spodyneiko'18).
- Taking $\nu = \eta R^{-2}$ with $\eta \to 0$, we find the RG flow in the undeformed limit

$$\frac{dR^2}{dt} = -(N - 2m - 2)R^2\,.$$

- Solving the renormalisation group flow equations for real η we find cyclic solutions. This motivates us to consider the analytically-continued regime

$$\nu \to i\nu\,, \quad \eta \to i\kappa\,,$$

  in which we have ancient solutions. In this regime the solution is

$$\nu = \text{constant}\,, \quad \kappa = -\tanh\left(\nu(N - 2m - 2)t\right)\,.$$

- Therefore the model in question is asymptotically free in the UV for $N - 2m > 2$. From now on we will concentrate on the simplest case of this type, i.e. $N = 5$ and $m = 1$ or $OSp(5|2)$.

$OSp(N|2m)$ action from $O(N+2m)$ action

- Although the general form of this trick is known to us, for conciseness let us consider the case $N = 2n+1$ and $m = 1$. The simplest way to write the deformed $O(2n+1)/O(2n)$ action is to use "stereographic" coordinates

$$ds^2 = \sum_{k=1}^{n} \frac{\kappa_k}{\nu} \frac{dz_k d\bar{z}_k}{(1+z_k\bar{z}_k)^2 \left(1 - \kappa_k^2 \left(\frac{1-z_k\bar{z}_k}{1+z_k\bar{z}_k}\right)^2\right)} \ ,.$$

  where

$$\kappa_k = \kappa \prod_{j=1}^{k-1} \left(\frac{1-z_j\bar{z}_j}{1+z_j\bar{z}_j}\right)^2 \ , \quad k = 1, \ldots, n \ .$$

- The transition to different deformations $OSp(N|2)$ action from the $O(N+2)$ is made by the substitution for some z_k

$$z_k \to \frac{\psi}{\sqrt{2}} = \frac{\psi^1 + i\psi^2}{\sqrt{2}} \ , \quad \bar{z}_k \to \frac{\bar{\psi}}{\sqrt{2}} = \frac{\psi^1 - i\psi^2}{\sqrt{2}} \ .$$

  Further we concentrate on the case $k = 2$.

- Also we go back to the "spherical" parametrization of the coordinates z_j

$$z_j = \sqrt{2\frac{1-r_j}{1+r_j}} e^{i\phi_j} \ .$$

The deformed $OSp(5|2)$ sigma model action

- Let us now turn to the specific case $OSp(5|2)$. The deformed sigma model is parametrised by four bosons, ϕ_1, ϕ_2, r_1 and r_2, and a symplectic fermion, ψ^a, where $a = 1, 2$.

- The Lagrangian following from the previous slide is

$$\mathcal{L}_\kappa^{(i)} = \frac{\kappa}{\nu(1-\kappa^2 r_1^2)} \left[\frac{\partial_+ r_1 \partial_- r_1}{1-r_1^2} + (1-r_1^2)\partial_+\phi_1\partial_-\phi_1 + \right.$$
$$\left. + i\kappa r_1(\partial_+ r_1 \partial_-\phi_1 - \partial_+\phi_1\partial_- r_1) \right] + \frac{\kappa r_1^2(1-\kappa^2 r_1^4 r_2^2 + (1+\kappa^2 r_1^4 r_2^2)\psi\cdot\psi)}{\nu(1-\kappa^2 r_1^4 r_2^2)^2} \times$$
$$\times \left[\frac{\partial_+ r_2 \partial_- r_2}{1-r_2^2} + (1-r_2^2)\partial_+\phi_2\partial_-\phi_2 + i\kappa r_1^2 r_2(1+\psi\cdot\psi)(\partial_+ r_2\partial_-\phi_2 - \partial_+\phi_2\partial_- r_2) \right] -$$
$$- \frac{\kappa r_1^2(1-\kappa^2 r_1^4 + \frac{1}{2}(1+\kappa^2 r_1^4)\psi\cdot\psi)}{\nu(1-\kappa^2 r_1^4)^2} \left[\partial_+\psi\cdot\partial_-\psi - i\kappa r_1^2(1+\tfrac{1}{2}\psi\cdot\psi)\partial_+\psi\wedge\partial_-\psi \right] ,$$

where we have introduced the following contractions of the symplectic fermion

$$\chi\cdot\chi' = \epsilon_{ab}\chi^a\chi'^b , \quad \chi\wedge\chi' = \delta_{ab}\chi^a\chi'^b .$$

UV limit of the deformed OSp(5|2) sigma model

- We are interested in the expansion around the UV fixed point, that is $\kappa = 1$. The specific limit we consider (Litvinov'18) is given by first setting

$$r_1 = \exp(-\epsilon e^{-2x_1}) , \quad r_2 = \tanh x_2 , \quad \psi^a = 2\epsilon^{\frac{1}{2}}\theta^a , \quad \kappa = 1 - \frac{\epsilon^2}{2} ,$$

and subsequently expanding around $\epsilon = 0$.

- Introducing the complex fields

$$X_1 = x_1 - i\phi_1 , \qquad X_2 = x_2 - i\phi_2 , \qquad \Theta = \theta^1 - i\theta^2 ,$$

we find the following expansion

$$\begin{aligned}\mathcal{L}^{(i)}_{\kappa\sim 1} = \frac{1}{\nu}\big(\partial_+ X_1 \partial_- X_1^* + \partial_+ X_2 \partial_- X_2^* + i e^{2x_1}(1 - i e^{2x_1}\Theta\Theta^*)\partial_+\Theta\partial_-\Theta^*\big) - \\ - \frac{\epsilon}{\nu}\big(e^{2x_1}\partial_+ X_1\partial_- X_1^* + e^{-2x_1+2x_2}(1 + 2ie^{2x_1}\Theta\Theta^*)\partial_+ X_2\partial_- X_2^* \\ + e^{-2x_1-2x_2}(1 + 2ie^{2x_1}\Theta\Theta^*)\partial_+ X_2^*\partial_- X_2 + \\ + \tfrac{i}{4}e^{4x_1}(1 - 2ie^{2x_1}\Theta\Theta^*)\partial_+\Theta\partial_-\Theta^*\big) + \mathcal{O}(\epsilon^2) ,\end{aligned}$$

up to total derivatives.

CFT's defined by screening charges

- Let $\boldsymbol{\varphi}(z) = (\varphi_1(z), \dots, \varphi_N(z))$ be the $N-$component holomorphic bosonic field normalized as

$$\varphi_i(z)\varphi_j(z') = -\delta_{ij}\log(z-z') + \dots \quad \text{at} \quad z \to z',$$

and $\vec{\boldsymbol{\alpha}} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_N)$ be the set of linear independent vectors.

- We define $W_{\vec{\alpha}}$-algebra as a set of currents $W_s(z)$ of integer spins s such that

$$\oint_{\mathcal{C}_z} e^{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi}(\xi))} W_s(z) d\xi = 0\,, \quad r = 1, \dots, N\,.$$

- For generic $\vec{\alpha}$ there is a spin 2 current

$$W_2(z) = -\frac{1}{2}(\partial\boldsymbol{\varphi}(z)\cdot\partial\boldsymbol{\varphi}(z)) + (\boldsymbol{\rho}\cdot\partial^2\boldsymbol{\varphi}(z))\,, \quad \boldsymbol{\rho} = \sum_{r=1}^{N}\left(1 + \frac{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)}{2}\right)\hat{\boldsymbol{\alpha}}_r\,,$$

and $(\boldsymbol{\alpha}_r \cdot \hat{\boldsymbol{\alpha}}_s) = \delta_{r,s}$. The corresponding central charge is

$$c = N + 12(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\,.$$

- For $N = 1$ we have a current

$$T(\varphi) = -\frac{1}{2}(\partial\varphi)^2 + \left(\frac{1}{\alpha} + \frac{\alpha}{2}\right)\partial^2\varphi\,.$$

The same algebra can be defined through the dual screening charge $\oint e^{\alpha^\vee \varphi} dz$ with $\alpha^\vee = \frac{2}{\alpha}$.

Bosonic and fermionic roots

- Depiction of bosonic roots

$$\bigcirc \text{ – bosonic root: } (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r) = \text{generic}$$

- If the current W_s satisfies commutativity condition it should be of a special form

$$W_s = W_s\Big(\mathsf{T}(\varphi_{\parallel}), \boldsymbol{\varphi}_{\perp}\Big)\,,$$

  where

$$\varphi_{\parallel} \overset{\text{def}}{=} \frac{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi})}{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)^{\frac{1}{2}}}, \quad \boldsymbol{\varphi}_{\perp} \overset{\text{def}}{=} \boldsymbol{\varphi} - \frac{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi})}{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)}\boldsymbol{\alpha}_r\,,$$

  and $\mathsf{T}(\varphi_{\parallel})$ is given by $W_2(z)$ with $\alpha = (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)^{\frac{1}{2}}$.
- Depiction of fermionic roots

$$\bigotimes \text{ – fermionic root: } (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r) = -1$$

- In the coordinates defined above it corresponds to the complex fermion. The communant of the corresponding screening charge $\oint e^{-i\varphi_{\parallel}(z)}dz$ consists of all $w_s = \psi^+\partial^{s-1}\psi$, $s = 2, 3, \dots$
- Among these currents only w_2 and w_3 are independent. Therefore

$$W_s = W_s\Big(w_2(\varphi_{\parallel}), w_3(\varphi_{\parallel}), \boldsymbol{\varphi}_{\perp}\Big). \tag{2.1}$$

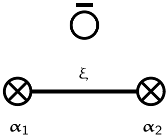
Properties of the systems with bosonic/fermionic roots

- **Bosonic root duality:** the bosonic roots always appear in pairs

$$\boldsymbol{\alpha} \quad \text{and} \quad \boldsymbol{\alpha}^{\vee} = \frac{2\boldsymbol{\alpha}}{(\boldsymbol{\alpha}\cdot\boldsymbol{\alpha})} \,.$$

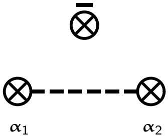
- **Dressed/sigma-model bosonic screening:** $(\boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) = \xi$ is arbitrary

$$\mathcal{S}_{\mathrm{B}} = \oint (\boldsymbol{\alpha}_1 \cdot \partial \boldsymbol{\varphi}) e^{(\boldsymbol{\beta}_{12}\cdot\boldsymbol{\varphi})} dz, \quad \text{where} \quad \boldsymbol{\beta}_{12} = \frac{2(\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2)}{(\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2)^2}$$



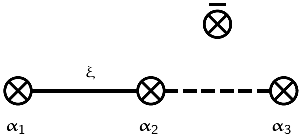
- **Dressed/sigma-model fermionic screening:** $(\boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) = -1$

$$\mathcal{S}_{\mathrm{F}} = \oint (\boldsymbol{\alpha}_1 \cdot \partial \boldsymbol{\varphi}) e^{(\boldsymbol{\beta}_{12}\cdot\boldsymbol{\varphi})} dz, \quad \text{where} \quad \boldsymbol{\beta}_{12} = \nu\boldsymbol{\alpha}_1 - (1+\nu)\boldsymbol{\alpha}_2$$



Dressed/sigma-model fermionic screening

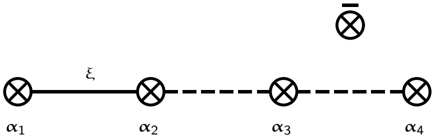
- The parameter ν cannot be fixed if only the two roots α_1 and α_2 are present.
- One way to fix the parameter ν is to embed in larger diagram. For example, consider the diagram



Then the parameter ν in the vector $\boldsymbol{\beta}_{23}$ is fixed from the condition

$$(\boldsymbol{\beta}_{23} \cdot \boldsymbol{\alpha}_1) = -1 \quad \Longrightarrow \quad \nu = -\frac{1}{\xi}.$$

- Another case also important for us is

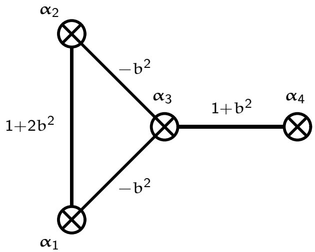


Then the parameter ν in the vector $\boldsymbol{\beta}_{34}$ is fixed from the condition

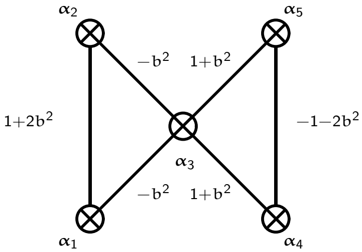
$$(\boldsymbol{\beta}_{34} \cdot \boldsymbol{\alpha}_2) = 1 - \xi \quad \Longrightarrow \quad \nu = \xi - 1.$$

Deformed O(5) sigma-model

- Our CFT $\frac{\widehat{\mathfrak{so}}(5)_{b^2-3}}{\widehat{\mathfrak{so}}(4)_{b^2-3}}$ with the central charge $c = 4 + \frac{30}{b^2} - \frac{12}{b^2}$ corresponds to the following diagram

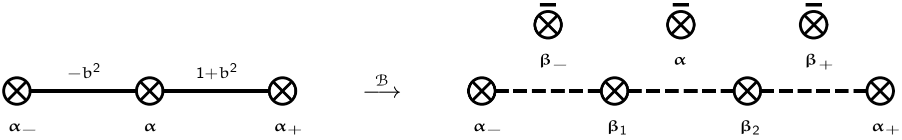


- Affinization of the diagram above corresponds to adding one root α_5 which completes triangle on the right



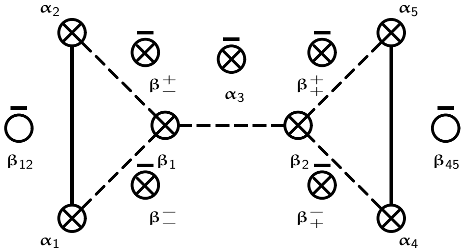
Blow-up transformation

- Now we describe transformation $\mathcal{B}$ of the root system, we call it *blow-up*, which acts as
$$O(N) \to OSP(N|2)\ ,$$
or more generally as
$$OSP(N|2m) \to OSP(N|2m+2)\ .$$
It can be applied to both conformal diagram and its affine counterpart.
- It acts on any root except $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, $\boldsymbol{\alpha}_{2n}$ and $\boldsymbol{\alpha}_{2n+1}$ and produces two fermionic roots out of one. On fermionic root $\boldsymbol{\alpha}$ it acts as follows
$$\boldsymbol{\alpha} = -b\mathbf{E} + i\beta\,\boldsymbol{e} \xrightarrow{\mathcal{B}} \{\boldsymbol{\beta}_1, \boldsymbol{\beta}_2\} = \left\{-\frac{1}{b}\mathbf{E} + \frac{i\beta}{b}\boldsymbol{\epsilon}, \frac{ib}{\beta}\boldsymbol{\epsilon} - \frac{i}{\beta}\boldsymbol{e}\right\}\ ,$$
where $\boldsymbol{\epsilon}$ is a new basic vector.
- Altogether this can be shown as follows



Screening charges for the deformed $OSp(5|2)$ sigma model

- Consider the simplest case of $OSP(5|2)$ affine diagram. According to our rule it is obtained from $O(5)$ diagram by blowing up the root $\boldsymbol{\alpha}_3$



- The vectors $\boldsymbol{\alpha}_r$ can be parameterized as follows ($\beta = \sqrt{1+b^2}$)

$$\boldsymbol{\alpha}_1 = b\mathbf{E}_1 + i\beta\,\boldsymbol{e}_1\,, \quad \boldsymbol{\alpha}_2 = b\mathbf{E}_1 - i\beta\,\boldsymbol{e}_1\,, \quad \boldsymbol{\alpha}_3 = -b\mathbf{E}_1 + i\beta\,\boldsymbol{e}_2\,,$$
$$\boldsymbol{\alpha}_4 = b\mathbf{E}_2 - i\beta\,\boldsymbol{e}_2\,, \quad \boldsymbol{\alpha}_5 = -b\mathbf{E}_2 - i\beta\,\boldsymbol{e}_2\,,$$
$$\boldsymbol{\beta}_1 = -\frac{1}{b}\mathbf{E}_1 + \frac{i\beta}{b}\boldsymbol{\epsilon}\,, \quad \boldsymbol{\beta}_2 = \frac{ib}{\beta}\boldsymbol{\epsilon} - \frac{i}{\beta}\boldsymbol{e}_2\,, \quad \boldsymbol{\beta}^{\pm}_{-} = \pm\frac{i}{\beta}\boldsymbol{e}_1 - \frac{ib}{\beta}\boldsymbol{\epsilon}\,,$$
$$\boldsymbol{\beta}^{\pm}_{+} = \pm\frac{1}{b}\mathbf{E}_2 - \frac{i\beta}{b}\boldsymbol{\epsilon}\,, \quad \boldsymbol{\beta}_{12} = \frac{1}{b}\mathbf{E}_1\,, \quad \boldsymbol{\beta}_{45} = \frac{i}{\beta}\boldsymbol{e}_2\,.$$

Dual model lagrangian

- In our case, there are two types of fields which cause UV divergencies. Either exponential ones

$$e^{(\boldsymbol{\alpha}\cdot\boldsymbol{\varphi})},$$

  or dressed/sigma-model fields

$$e^{(\boldsymbol{\beta}\cdot\boldsymbol{\varphi})}(\boldsymbol{\alpha},\partial\boldsymbol{\varphi})(\boldsymbol{\alpha}^*,\bar{\partial}\boldsymbol{\varphi}),\quad (\boldsymbol{\alpha},\boldsymbol{\alpha})=-1,\quad (\boldsymbol{\alpha},\boldsymbol{\beta})=(\boldsymbol{\alpha}^*,\boldsymbol{\beta})=1.$$

- OPE of exponential fields has the form

$$e^{(\boldsymbol{\alpha}_r\cdot\boldsymbol{\varphi}(z))}e^{(\boldsymbol{\alpha}_s\cdot\boldsymbol{\varphi}(w))}=\left|\frac{r_0}{z-w}\right|^{2(\boldsymbol{\alpha}_r\cdot\boldsymbol{\alpha}_s)}e^{((\boldsymbol{\alpha}_r+\boldsymbol{\alpha}_s)\cdot\boldsymbol{\varphi}(w))}+\dots$$

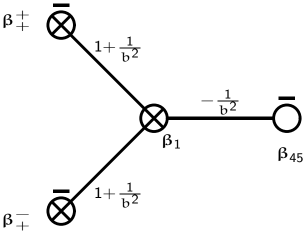
- We see that the perturbation theory contains divergent integrals for all scalar products which tend to 1 in the limit $b\to 0$. These UV divergences can be regularized by subtracting the following counter terms from the Lagrangian

$$\frac{\pi\Lambda_r\Lambda_s r_0^{(\boldsymbol{\alpha}_r+\boldsymbol{\alpha}_s)^2}}{(\boldsymbol{\alpha}_r\cdot\boldsymbol{\alpha}_s)-1}e^{((\boldsymbol{\alpha}_r+\boldsymbol{\alpha}_s)\cdot\boldsymbol{\varphi})},$$

  for each $\boldsymbol{\alpha}_r$ and $\boldsymbol{\alpha}_s$ such that $(\boldsymbol{\alpha}_r\cdot\boldsymbol{\alpha}_s)\to 1$ in the limit $b\to 0$.

Metric for the deformed OSp(5|2) sigma model

- By taking the dual screenings we obtain the following system, which includes the dressed screenings



- By choosing $z = x^1 - ix^2$ ($\bar{z} = x^1 + ix^2$) and then conducting Wick rotation $x^2 = ix^0$, we obtain the action in Minkowski signature

$$\mathcal{L} = \frac{1}{8\pi}\left(\sum_{i=1}^{2}(\partial_+\Phi_i)(\partial_-\Phi_i) + \sum_{j=1}^{3}(\partial_+\phi_j)(\partial_-\phi_j)\right) +$$

$$+\Lambda_1 e^{-\frac{i\beta}{b}\phi_3}\left(\partial_+(b\Phi_2 + i\beta\phi_2)\,\partial_-(b\Phi_2 - i\beta\phi_2)\,e^{-\frac{\Phi_2}{b}} +\right.$$

$$\left.+\partial_+(b\Phi_2 - i\beta\phi_2)\,\partial_-(b\Phi_2 + i\beta\phi_2)\,e^{\frac{\Phi_2}{b}}\right) + \Lambda_2 e^{-\frac{\Phi_1}{b} + \frac{i\beta}{b}\phi_3} +$$

$$+\Lambda_3\partial_+(b\Phi_1 + i\beta\phi_1)\,\partial_-(b\Phi_1 - i\beta\phi_1)\,e^{\frac{\Phi_1}{b}} + \frac{\pi b^2}{\beta^2}\Lambda_1\Lambda_2 e^{\frac{\Phi_1}{b}} \times$$

$$\times\left(\partial_+(b\Phi_2 + i\beta\phi_2)\,\partial_-(b\Phi_2 - i\beta\phi_2)\,e^{-\frac{\Phi_2}{b}} + \partial_+(b\Phi_2 - i\beta\phi_2)\,\partial_-(b\Phi_2 + i\beta\phi_2)\,e^{\frac{\Phi_2}{b}}\right) + \ldots,$$

Restoring the deformed OSp(5|2) sigma model in the UV limit

- Then we fermionize the ϕ_3 field This after the integrations over the Ψ_1 and $\Psi_2^\dagger$ components yields the following action

$$\mathcal{L} = \frac{1}{8\pi}\left(\sum_{i=1}^{2}(\partial_+\Phi_i)(\partial_-\Phi_i) + \sum_{j=1}^{2}(\partial_+\phi_j)(\partial_-\phi_j)\right) + 2i\Psi_1^\dagger\partial_-\Psi_1 + 2i\Psi_2^\dagger\partial_+\Psi_2 +$$

$$+\frac{2\pi}{\beta^2}\Psi_1^\dagger\Psi_2^\dagger\Psi_2\Psi_1 - i\Lambda_1\Psi_1^\dagger\Psi_2 e^{-\frac{i\beta}{b}\phi_3}\left(\partial_+\left(b\Phi_2 + i\beta\phi_2\right)\partial_-\left(b\Phi_2 - i\beta\phi_2\right)e^{-\frac{\Phi_2}{b}} +\right.$$

$$\left. +\partial_+\left(b\Phi_2 - i\beta\phi_2\right)\partial_-\left(b\Phi_2 + i\beta\phi_2\right)e^{\frac{\Phi_2}{b}}\right) - i\Lambda_2\Psi_1\Psi_2^\dagger e^{-\frac{\Phi_1}{b}} +$$

$$+\Lambda_3\partial_+\left(b\Phi_1 + i\beta\phi_1\right)\partial_-\left(b\Phi_1 - i\beta\phi_1\right)e^{\frac{\Phi_1}{b}} + \frac{\pi b^2}{\beta^2}\Lambda_1\Lambda_2 e^{\frac{\Phi_1}{b}}\times$$

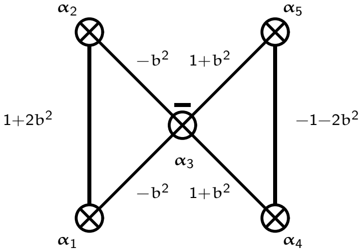
$$\times\left(\partial_+\left(b\Phi_2 + i\beta\phi_2\right)\partial_-\left(b\Phi_2 - i\beta\phi_2\right)e^{-\frac{\Phi_2}{b}} +\right.$$

$$\left. +\partial_+\left(b\Phi_2 - i\beta\phi_2\right)\partial_-\left(b\Phi_2 + i\beta\phi_2\right)e^{\frac{\Phi_2}{b}}\right) + \ldots,$$

- This after the integrations over the Ψ_1 and $\Psi_2^\dagger$ upon identifying $\Phi_{1,2} = 2bx_{2,1}$, $\phi_{1,2} = 2b\varphi_{2,1}$ and $\Psi_1^\dagger = b\Theta^*$, $\Psi_2 = b\Theta$ together with taking the limit $b \to \infty$ and adjusting properly the coefficients $\Lambda_{1,2,3}$ ($\alpha' = \frac{2}{b^2}$) we obtain dividing by 4 the UV limit originating from the screening picture.

Screening charges in the $b \to 0$ limit

- By taking the subsystem of screenings, which are regular in the limit $b \to 0$



- We are able to write the lagrangian of the dual model

$$\mathcal{L} = \frac{1}{8\pi}\left(\sum_{i=1}^{2}(\partial\Phi_i)(\bar\partial\Phi_i) + \sum_{j=1}^{3}(\partial\varphi_j)(\bar\partial\varphi_j)\right) + 2\Lambda_1 e^{b\Phi_1}\cos\beta\varphi_1 +$$
$$+\Lambda_2\partial(\Phi_1 - i\beta\varphi_3)\bar\partial(\Phi_1 + i\beta\varphi_3)e^{-b\Phi_1+i\beta\varphi_2}+$$
$$+\Lambda_3\left(e^{-b\Phi_2-i\beta\varphi_2}+e^{b\Phi_2-i\beta\varphi_2}\right)+(\text{counterterms})$$

- This action appears to have only finite number of counterterms!

Dual model lagrangian for the OSp(5|2) case

- Utilizing the bosonization of the complex fermion and $\beta\gamma$ system

$$e^{b\Phi_1} \to \bar{\beta}\beta\,, \quad \left(\frac{1}{b}\partial\Phi_1 + \frac{i\beta}{b}\partial\varphi_3\right)\left(\frac{1}{b}\bar{\partial}\Phi_1 - \frac{i\beta}{b}\bar{\partial}\varphi_3\right)e^{-b\Phi_1} \to \bar{\gamma}\gamma\,,$$

we get after rescaling $\Phi_2 = 2\sqrt{\pi}\Phi$ and $\hat{b} = 2\sqrt{\pi}b$

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial\Phi\bar{\partial}\Phi + i\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\bar{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \\ &+ \frac{m^2}{2\hat{b}^2}\cosh^2\hat{b}\Phi + m\bar{\Psi}_1\Psi_1\cosh\hat{b}\Phi + m\bar{\Psi}_2\Psi_2\cosh\hat{b}\Phi + m(\bar{\beta}\beta - \bar{\gamma}\gamma)\cosh\hat{b}\Phi \\ &- \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_1\gamma^\mu\Psi_1)^2 - \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_2\gamma^\mu\Psi_2)^2 - \frac{4\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}\bar{\beta}\beta\bar{\gamma}\gamma + \frac{\hat{b}^2}{2}(\bar{\gamma}\gamma)^2 + \\ &+ \hat{b}^2\bar{\Psi}_1\Psi_1\bar{\Psi}_2\gamma_+\Psi_2 + \hat{b}^2\bar{\Psi}_1\Psi_1\bar{\gamma}\gamma + \hat{b}^2\bar{\Psi}_2\gamma_+\Psi_2(\bar{\beta}\beta - \bar{\gamma}\gamma)\,,\end{aligned}$$

- Integrating out β and putting $\gamma = \sqrt{m}\Upsilon$

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial\Phi\bar{\partial}\Phi + i\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\bar{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 + \\ &+ \left(\cosh\hat{b}\Phi + \frac{4\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}\bar{\Upsilon}\Upsilon - \hat{b}^2 m^{-1}\bar{\Psi}_2\gamma_+\Psi_2\right)^{-1}\partial\bar{\Upsilon}\bar{\partial}\Upsilon + \\ &+ \frac{m^2}{2\hat{b}^2}\cosh^2\hat{b}\Phi + m\bar{\Psi}_1\Psi_1\cosh\hat{b}\Phi + m\bar{\Psi}_2\Psi_2\cosh\hat{b}\Phi + m^2\bar{\Upsilon}\Upsilon\cosh\hat{b}\Phi \\ &- \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_1\gamma^\mu\Psi_1)^2 - \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_2\gamma^\mu\Psi_2)^2 + \frac{\hat{b}^2}{2}m^2(\bar{\Upsilon}\Upsilon)^2 + \\ &+ \hat{b}^2\bar{\Psi}_1\Psi_1\bar{\Psi}_2\gamma_+\Psi_2 - \hat{b}^2 m\bar{\Psi}_1\Psi_1\bar{\Upsilon}\Upsilon + \hat{b}^2 m\bar{\Psi}_2\gamma_+\Psi_2\bar{\Upsilon}\Upsilon\,.\end{aligned}$$

Wick rotation and tree-level S-matrix

- Now we are to continue to the Lorentzian signature

$$\mathcal{L} = \frac{1}{2}\partial_-\Phi\partial_+\Phi + i\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\bar{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 +$$
$$+ \left(\cosh\hat{b}\Phi + \frac{4\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}\bar{\Upsilon}\Upsilon + \hat{b}^2 m^{-1}\bar{\Psi}_2\gamma_+\Psi_2\right)^{-1}\partial_-\bar{\Upsilon}\partial_+\Upsilon$$
$$- \frac{m^2}{2\hat{b}^2}\cosh^2\hat{b}\Phi - m\bar{\Psi}_1\Psi_1\cosh\hat{b}\Phi - m\bar{\Psi}_2\Psi_2\cosh\hat{b}\Phi - m^2\bar{\Upsilon}\Upsilon\cosh\hat{b}\Phi$$
$$- \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_1\gamma^\mu\Psi_1)^2 - \frac{\hat{b}^2}{8+\frac{2}{\pi}\hat{b}^2}(\bar{\Psi}_2\gamma^\mu\Psi_2)^2 - \frac{\hat{b}^2}{2}m^2(\bar{\Upsilon}\Upsilon)^2$$
$$- \hat{b}^2\bar{\Psi}_1\Psi_1\bar{\Psi}_2\gamma_+\Psi_2 + \hat{b}^2 m\bar{\Psi}_1\Psi_1\bar{\Upsilon}\Upsilon - \hat{b}^2 m\bar{\Psi}_2\gamma_+\Psi_2\bar{\Upsilon}\Upsilon\,,$$

  which allows us to write the lagrangian in the 1-loop approximation

$$\mathcal{L} = \frac{\partial_-\Phi\partial_+\Phi}{2} - \frac{m^2}{2}\Phi^2 + \bar{\Psi}_1(i\gamma^\mu\partial_\mu - m)\Psi_1 + \bar{\Psi}_2(i\gamma^\mu\partial_\mu - m)\Psi_2 + \partial_-\bar{\Upsilon}\partial_+\Upsilon - m^2\bar{\Upsilon}\Upsilon$$
$$- \frac{\hat{b}^2}{6}m^2\Phi^4 - \frac{\hat{b}^2}{2}m\bar{\Psi}_1\Psi_1\Phi^2 - \frac{\hat{b}^2}{2}m\bar{\Psi}_2\Psi_2\Phi^2 - \frac{\hat{b}^2}{2}(\partial_-\bar{\Upsilon}\partial_+\Upsilon + m^2\bar{\Upsilon}\Upsilon)\Phi^2$$
$$- \frac{\hat{b}^2}{8}(\bar{\Psi}_1\gamma^\mu\Psi_1)^2 - \frac{\hat{b}^2}{8}(\bar{\Psi}_2\gamma^\mu\Psi_2)^2 - \frac{\hat{b}^2}{2}\bar{\Upsilon}\Upsilon(\partial_-\bar{\Upsilon}\partial_+\Upsilon + m^2\bar{\Upsilon}\Upsilon)$$
$$- \hat{b}^2\bar{\Psi}_1\Psi_1\bar{\Psi}_2\gamma_+\Psi_2 + \hat{b}^2 m\bar{\Psi}_1\Psi_1\bar{\Upsilon}\Upsilon - \hat{b}^2 m^{-1}\bar{\Psi}_2\gamma_+\Psi_2(\partial_-\bar{\Upsilon}\partial_+\Upsilon + m^2\bar{\Upsilon}\Upsilon)\,,$$

- We checked that the $2 \to 2$ tree-level S-matrix of the obtained lagrangian satisfies the classical Yang-Baxter equation upon identification $\lambda = \frac{1}{2} - \frac{b^2}{2} + \mathcal{O}(b^4)$ together with some gauge and twist transformation.

Conclusions and outlook

- We found the action of the η-deformed $OSp(N|2m)$ sigma models for several N and m and put forward the hypothesis how to generate this action for general N and m.
- The 1-loop RG flow of such models was studied and we found the UV stable solutions. We considered the scaling limit of the deformed $OSp(5|2)$ sigma model action as an example.
- The system of screening charges, which determine the integrable structure of the $OSp(N|2)$ sigma model was built.
- By using it we demonstrated how to restore the sigma model action in the deep UV in the case of $OSp(5|2)$.
- Utilizing our system of screenings to write the dual model with the Toda type interactions we can reproduce the expansion of the S-matrix in the vicinity of the special point $\lambda = \frac{1}{2}$, checking that it satisfies the classical Yang-Baxter equation.
- The next interesting step would be to try to adapt the dual description for the sigma models with the non-compact target space (Basso, Zhong'18).

Thanks for your attention!